Eg 5 (1980)

8. A particle is moving in a straight line such that its distance x from a fixed point at time t is given by

$$x = r\cos \omega t .$$

Show that the particle is moving with simple harmonic motion.
A particle is moving in a straight line with simple harmonic motion. When it is a point p_1 of distance 0·8 m from the mean-centre, its speed is 6 m/s and when it is at a point p_2 of distance 0·2 m from the end-poisition on the same side of the mean-centre as p_1, its acceleration is of magnitude 24 m/s². If r is the amplitude of the motion, show that

$$\frac{2}{3} = \frac{r - 0{\cdot}2}{r^2 - 0{\cdot}64}$$

and hence find the value of r.

Find also the period of the motion and the shortest time taken between p_1 and p_2 correct to two places of decimals.

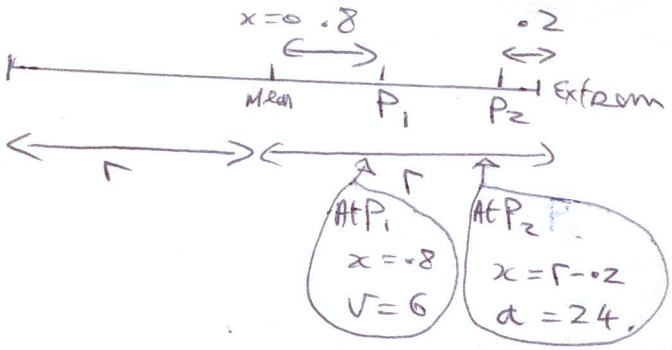
$x = r\cos\omega t$.

Show SHM: $\frac{dx}{dt} = -r\omega \sin \omega t$.

$\frac{d^2x}{dt^2} = -\omega^2(r\cos\omega t)$

$\frac{d^2x}{dt^2} = -\omega^2 x$

$\Rightarrow a = -\omega^2 x$ as required for SHM.



$\therefore$ At p_1 $v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow 6^2 = \omega^2(r^2 - (.8)^2)$

$\Rightarrow 36 = \omega^2(r^2 - 0{\cdot}64)$ (1)

At p_2: $|a| = \omega^2 x$

$\Rightarrow 24 = \omega^2(r - .2)$ (2)

Solve $\frac{(2)}{(1)} \Rightarrow \frac{24}{36} = \frac{\omega^2(r-.2)}{\omega^2(r^2-.64)}$

$\Rightarrow \boxed{\frac{2}{3} = \frac{r-.2}{r^2-.64}}$ qed

To find r: $\Rightarrow 2(r^2 - .64) = 3(r - .2)$

$\Rightarrow 2r^2 - 1{\cdot}28 = 3r - .6$

$\Rightarrow 2r^2 - 3r - .68 = 0$

$\Rightarrow r = -.2$ or $r = 1{\cdot}7$.

$r = -.2$ has no meaning here so pack

$\boxed{r = 1{\cdot}7}$

Next find T: first ω

(2) $\Rightarrow 24 = \omega^2(1{\cdot}7 - .2)$

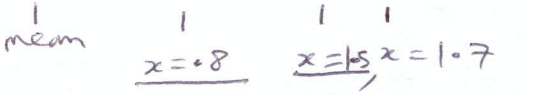
$24 = \omega^2(1{\cdot}5)$

$16 = \omega^2$

$\boxed{4 = \omega}$

$\therefore T = \frac{2\pi}{4} = \frac{\pi}{2}$ seconds

Next time to travel $[P_1P_2]$



$t_{P_1P_2} = t_{OP_2} - t_{OP_1}$

t_{OP_1}: Use $x = A\sin\omega t$

$.8 = 1{\cdot}7\sin 4t$.

$.4706 = \sin 4t$.

$.49 = 4t$

$\boxed{.1225 = t_{OP_1}}$

t_{OP_2}: $x = A\sin\omega t$

$1{\cdot}5 = 1{\cdot}7\sin 4t$.

$.8824 = \sin 4t$

$1{\cdot}0809 = 4t$

$\boxed{0{\cdot}2702 = t_{OP_2}}$

$\boxed{t_{P_1P_2} = .2702 - .1225 = .1477 \text{ sec}}$